Aircraft Maintenance Jacking Problem via Optimization

Marcel Mongeau, Christian Bès

Abstract

We address the problem of assigning forces to jacking positions in order to weaken stress at points where an aircraft maintenance operation has to be performed. We introduce a mixed-integer linear programming model, and report encouraging computational experiments on historical data. Our methodology is currently under the process of industrial implementation at Airbus, where it will be used as a maintenance decision-analysis tool.

I. Introduction

In order to perform structural or other maintenance interventions on an aircraft (such as repairs, modifications, system checks), it is necessary that residual internal stresses be weak in the intervention zone (in order to enable dismantling or part assembling). This is generally not the case when the aircraft is only supported by its landing gears (or by the main jacks positioned near the landing gears). Thus one relies on using a certain number of auxiliary jacks. The aircraft maintenance jacking problem is concerned with assigning forces to be applied on various jacking positions in order to weaken stress at the points where the maintenance has to be performed.

More specifically, given a list of jacking locations (including, for each of them, its location on the aircraft and an upper bound on the force that can be applied on that zone of the aircraft), a list of control points where the residual internal stresses are measured, the distribution of the aircraft mass, we are concerned with the problem of deciding which of the available jacking locations should be selected, and what amount of force should be applied on the corresponding jacks, in order to achieve the above-mentioned weak residual internal stress condition. At the same time, the number of selected jacking locations must be as low as possible (in order to minimize time and operation costs).

In common practice, this problem is solved by trials and errors performed by an experienced engineer. For instance, Airbus currently uses an internal software to compute the residual internal stresses corresponding to a given list of ("guessed") jacking locations and corresponding forces (both proposed by an experienced engineer), a list of control points, and the distribution of the aircraft mass.

The only published literature we could find related to the aircraft maintenance jacking problem is [1] (and references therein) which presents an automated procedure to determine the jack loads for static and fatigue structural tests while providing a best fit to known internal moment, shear, and torsion loads. The aim for them is to reproduce the loads to which the structure will be subjected in service, whereas we want to minimize loads. The main difference with the problem we are considering lies in the inherent combinatorial nature of the maintenance jacking problem: we want to use as few jacks as possible (in order to expedite the maintenance operation). This is why the (purely continuous) least-squares approach introduced in [1] (where every available jacking position is used) is not suitable in our context.

In the current paper, we are proposing a *mixed integer linear programming* approach (see for instance [2]) to solve the aircraft maintenance jacking problem. The goal of this approach is to improve the residual internal stresses and to compute a solution faster than when using procedures based on trials and errors.

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M. Mongeau is with the Labo MIP, Université Paul Sabatier, Toulouse, France. E-mail: mongeau@mip.ups-tlse.fr

C. Bès, formerly with Airbus France, Toulouse, is now with the Dept Génie Mécanique, Université Paul Sabatier, Toulouse, France. E-mail: cbes@cict.fr

The paper is organized as follows. We introduce the model formulation in the next section, where we show how the jacking problem can be expressed as a linear programming problem with both continuous and binary variables. The binary variables tell which set of jacking locations are to be used, while the continuous variables give the force to be applied on each jacking location. In Section III, we report very encouraging computational experiments with the jacking problem for maintenance operations on Airbus A310, A300-B4, and A300-B2 aircrafts. We conclude in Section IV.

II. Model Formulation

For practical purposes, we assume that the airplane can be modelled by beams, and therefore "minimizing residual internal stresses" means minimizing the magnitudes of the bending momentum, the torque and the shear load, as measured at certain control points chosen by the user.

We consider a system of coordinates whose origin is placed at the nose of the aircraft. The first coordinate is along the longitudinal axis of the aircraft (positive towards aft), the second coordinate is transversal (positive towards left-hand side wing, negative towards right-hand side wing), and the third coordinate (when needed) is oriented to be positive towards the ground (see Figure 1).

A. Input

The data assumed to be given is denoted with upper-case letters as follows:

- N_J Number of available jacking locations,
- N Number of allowed jacks ($N \leq N_J$),
- N_C Number of control points,
- N_P Number of aircraft parts (discretized masses),
- W Weights: vector of aircraft mass distribution, ($\dim(W) = N_P$),
- L Vector of lower bounds,
- U Vector of upper bounds,
- Θ Sweepback angle (see Figure 1),

where the pth component of W, denoted by W_p, is the weight of part p ($p = 1, 2, \ldots, N_P$); the jth component of U, denoted by U_j, is the upper bound on the amount of force that can be applied on jacking location j ($j = 1, 2, \ldots, N_J$); and similarly for L, the vector of lower bounds. Naturally, we assume that

$$0 \leq L_j \leq U_j, \quad j = 1, 2, \ldots, N_J.$$

The remaining given data includes the positions (respectively, longitudinal and transverse coordinates) of the jacking locations, the control points, and the center of mass of aircraft parts:

- $X(j), Y(j)$ coordinates of the jth jacking location, $j = 1, 2, \ldots, N_J$,
- $X(c), Y(c)$ coordinates of the cth control point, $c = 1, 2, \ldots, N_C$,
- $X(p), Y(p)$ coordinates of the center of mass of the pth aircraft part, $p = 1, 2, \ldots, N_P$,

Although this may yield ambiguity, for the sake of simplicity, we use the same notation, $X(\cdot), Y(\cdot)$, for the three sets of coordinates. However, confusion will be avoided by using the following notational convention:

- an index j will always refer to a jacking location,
- an index c will always refer to a control point,
- an index p will always refer to a center of mass of an aircraft part.

B. Optimization variables

For each jacking location, $j = 1, 2, \ldots, N_J$, we define the following variables (lower-case letters are used for the unknowns to be optimized):

- y_j is 1 if jacking location number j is used, and 0 otherwise,
- x_j is the force to be applied on jacking location number j, e.g. in Newton (N).

C. Further notation

We now define notation which we shall use in in order to define our objective function. These quantities results from the forces x_j's (to be determined) applied on the various jacking locations j ($j = 1, 2, \ldots, N_J$), and also from the (given) aircraft mass distribution (the weights W_p's of the different parts p of the aircraft, $p = 1, 2, \ldots, N_P$):

- $M_B^c(x)$ bending momentum at control point c,
- $M_T^c(x)$ torque at control point c,
- $S^c(x)$ shear load on control point c,
- $M_X^c(x)$ longitudinal (i.e. along the x-axis) momentum at control point c,
- $M_Y^c(x)$ transverse (i.e. along the y-axis) momentum at control point c.

Our aim will be to minimize a linear combination of $|M_B^c(x)|$, $|M_T^c(x)|$, and $|S^c(x)|$ ($c = 1, 2, \ldots, N_C$). When the control point c is located on the fuselage, we have

$$M_B^c(x) = M_Y^c(x) \quad \text{and} \quad M_T^c(x) = M_X^c(x). \tag{1}$$

However, when c is on the wings, for instance on the left-hand wing, the relation is

$$\begin{aligned} M_B^c(x) &= M_Y^c(x)\sin\Theta + M_X^c(x)\cos\Theta, \quad \text{and} \\ M_T^c(x) &= M_X^c(x)\sin\Theta + M_Y^c(x)\cos\Theta \end{aligned} \tag{2}$$

(see Figure 1). Analogous relations must be derived when there is a second sweepback angle involved in the wings.

In order to define the momenta $M_X^c(x)$ and $M_Y^c(x)$, and the shear load $S^c(x)$ (in Section II-D) in terms of the given data and the optimization variables, let us first introduce some notations which will prove convenient. We first define the sign function:

$$\operatorname{sign}(x) = \begin{cases} 1 & \text{if } x > 0, \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0, \end{cases}$$

and the "+" function:

$$x^+ = \begin{cases} x & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

The indicator function $1_{\{\text{condition}\}}$ (where "condition" is some logical condition), is defined as follows:

$$1_{\{\text{condition}\}} = \begin{cases} 1 & \text{if condition} \\ 0 & \text{otherwise.} \end{cases}$$

D. Modeling the momenta

In this section, we describe precisely in terms of the given data and the optimization variables, the expressions of the momenta and shear load: $M_X^c(x)$, $M_Y^c(x)$, and $S^c(x)$ (see e.g. [3]).

We assume that the aircraft is made of three rigid beams, one for the fuselage and two for the wings. Let us also assume, for now, that all the jacking locations are used (i.e. we disregard temporarily the binary variables y_j which monitor, for each jacking location j, whether it is used or not).

• When a vertical force of magnitude x_j is applied at a **jacking location** j (located at $X(j), Y(j)$), then the longitudinal and transverse momenta are respectively the first and the second components of the vector given by the cross product :

$$\begin{pmatrix} X(j) - X(c) \\ Y(j) - Y(c) \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -x_j \end{pmatrix}$$

(remember that by our choice of system of coordinates, the force applied on a jacking location must be negative, since it is oriented upwards, see Figure 1Thus, the *longitudinal* momentum (at control point c, resulting from the force x_j applied on jacking location j) is $[Y(c) - Y(j)]x_j$ and the *transverse* momentum (at control point c, resulting from the force x_j applied on jacking location j) is $[X(j) - X(c)]x_j$.

• Similarly, when the vertical force corresponds to the **weight** W_p of the pth aircraft part (whose center of mass is located at $X(p), Y(p)$), the longitudinal and transverse momenta (at control point c, resulting from the weight W_p) are respectively $[Y(p) - Y(c)]W_p$ and $[X(c) - X(p)]W_p$.

It will then remain to sum the contributions from each jacking location j, $j = 1, 2, \ldots, N_J$, and from each weight W_p, $p = 1, 2, \ldots, N_P$.

In order to ease the computation of the momenta and shear loads (in terms of the given data and the optimization variables x and y), we propose to decompose the aircraft into four components: fore fuselage, aft fuselage, left-hand side wing, and right-hand side wing. In both the cases of the force applied on a jacking location or of the weight of an aircraft part, we are concerned in this paper with *residual* internal stresses. Hence, because the aircraft is in equilibrium, in order to compute these expressions, we only take into account contributions from the cases where the jacking location (or the center of mass, respectively) is located *downstream* from control point c. (As it is equivalent for computing purposes, we can alternatively only take into account contributions from the cases where the jacking location (or the center of mass, respectively) is located *upstream* from control point c, when this facilitates the computation.)

More precisely, let us consider a particular control point c (of coordinates $X(c), Y(c)$). Here are the explicit expressions of $M_B^c(x)$, $M_T^c(x)$, and $S^c(x)$ in terms of the given data and the optimization variables, for each possible location of the control point c. If c is located on the:

1. Fuselage: Because each fuselage jacking location j and each center of mass of the aircraft part p are along the x axis by symmetry, and because of the aircraft equilibrium, we have that the longitudinal momentum reduces to zero:

$M_X^c(x) = 0$.

Fore fuselage: (upstream contributions)

$$M_Y^c(x) = \sum_{j=1}^{N_J} -x_j[X(c) - X(j)]^+ + \sum_{p=1}^{N_P} W_p[X(c) - X(p)]^+,$$

$$S^c(x) = \sum_{j=1}^{N_J} -x_j 1_{\{X(c) \geq X(j)\}} + \sum_{p=1}^{N_P} W_p 1_{\{X(c) \geq X(p)\}};$$

Aft fuselage: (downstream contributions)

$$M_Y^c(x) = \sum_{j=1}^{N_J} x_j[X(j) - X(c)]^+ + \sum_{p=1}^{N_P} -W_p[X(p) - X(c)]^+,$$

$$S^c(x) = \sum_{j=1}^{N_J} -x_j 1_{\{X(c) \leq X(j)\}} + \sum_{p=1}^{N_P} W_p 1_{\{X(c) \leq X(p)\}};$$

2. Wings:

Left-hand side wing: (downstream contributions)

$$M_X^c(x) = \sum_{j=1}^{N_J} -x_j[Y(j) - Y(c)]^+ + \sum_{p=1}^{N_P} W_p[Y(p) - Y(c)]^+,$$

$$M_Y^c(x) = \sum_{j=1}^{N_J} x_j[X(j) - X(c)] 1_{\{Y(j) \geq Y(c)\}} + \sum_{p=1}^{N_P} -W_p[X(p) - X(c)] 1_{\{Y(p) \geq Y(c)\}},$$

$$S^c(x) = \sum_{j=1}^{N_J} -x_j 1_{\{Y(j) \geq Y(c)\}} + \sum_{p=1}^{N_P} W_p 1_{\{Y(j) \geq Y(c)\}};$$

Right-hand side wing: We proceed in an analogous manner.

All the above expressions are composed of two main terms: the first one sums up the contributions from the forces x_j applied on each jacking location j, and the second term sums up the contribution from the weights W_p of each aircraft part p.

Finally, remark that all the above expressions are linear (or constant) in the optimization variable x. More precisely, we have

$$\begin{aligned} M_X^c(x) &= C_X(c)^T x + D_X(c), \\ M_Y^c(x) &= C_Y(c)^T x + D_Y(c), \\ S^c(x) &= C_S(c)^T x + D_S(c), \end{aligned}$$

for some vectors $C_X(\cdot), C_Y(\cdot), C_S(\cdot)$ and some scalars $D_X(\cdot), D_Y(\cdot), D_S(\cdot)$ which are functions of the given data: the control point c, the aircraft geometry, the aircraft mass distribution, and the jacking locations. Therefore, using the linear relations (2), we also have

$$\begin{aligned} M_B^c(x) &= C_B(c)^T x + D_B(c), \\ M_T^c(x) &= C_T(c)^T x + D_T(c), \\ S^c(x) &= C_S(c)^T x + D_S(c), \end{aligned} \tag{3}$$

for some vectors $C_B(\cdot), C_T(\cdot)$ and some scalars $D_B(\cdot), D_T(\cdot)$.

E. *Objective function*

We have two conflicting objectives. We want to minimize residual internal stresses at the control points, while using as few jacks as possible to achieve this aim. Let us continue (temporarily) to ignore the second objective. In other words, we still assume for a moment that all possible jacking locations can be used.

Minimizing residual internal stresses at the control points (with the beam hypothesis) corresponds to minimizing the magnitudes of the bending momentum, the torque, and the shear load: $|M_B^c(x)|$, $|M_T^c(x)|$, and $|S^c(x)|$, and this, at each control point, c, $c = 1, 2, \ldots, N_C$ (see [3]).

Two natural ways of doing are minimizing the sum of the absolute values, or alternatively, minimizing the sum of squares. Let us first introduce a weighting parameter λ ($0 \leq \lambda \leq 1$), whose value will be set by the user, because the momenta (bending momenta and torques) and the shear loads are quantities which are expressed in different units—Newtons $\cdot$ meters versus Newtons, for instance). Thus, we either consider minimizing

$$\sum_{c=1}^{N_C} \left(\frac{1-\lambda}{2} \left| M_B^c(x) \right| + \frac{1-\lambda}{2} \left| M_T^c(x) \right| + \lambda \left| S^c(x) \right| \right); \tag{4}$$

or, alternatively, we consider minimizing

$$\sum_{c=1}^{N_C} \left(\frac{1-\lambda}{2} \left(M_B^c(x) \right)^2 + \frac{1-\lambda}{2} \left(M_T^c(x) \right)^2 + \lambda \left(S^c(x) \right)^2 \right), \tag{5}$$

where, in both cases, $x \in I\!R^{N_J}$. One usually favours the least-squares formulation (5), as the absolute value formulation (4) involves a non-differentiable objective function. However, for our application, we prefer the absolute value formulation (4) so as to obtain by a simple transformation (see below) a linear objective function, and keep linear the sensitivity of the objective function. (This is not to mention the robustness advantage of the absolute value criterion over the least-squares criterion, see e.g. [4].) Indeed, remember that by (3), the three quantities $M_B^c(x)$, $M_T^c(x)$, and $S^c(x)$ are linear in x. Furthermore, one can easily show that the piecewise-linear (in terms of the optimization variable x) objective function (4) can be replaced by a *linear* objective function, provided that we add some linear constraints. Moreover, we are about to incorporate binary optimization variables (the y's monitoring which jacking locations are used) to our model, which, therefore will become a mixed-integer program. It is clear that the difficulty of solving a mixed integer *linear* programming problem is incomparably lower than that of solving the mixed integer *quadratic* programming problem which formulation (5) would yield.

Replacing a piecewise-linear optimization problem with a linear optimization problem is quite standard in applied mathematics (e.g. in order to solve l_1-fitting problems). In short, consider the general situation (to simplify the presentation):

$$\min_{u \in I\!R^n} \sum_{i=1}^{m} |a_i^T u + b_i|, \tag{6}$$

where m and n are given positive integers, and the a_i's, and the b_i's are given vectors of $I\!R^n$ and given scalars, respectively. Performing the change of variables:

$$v_i := |a_i^T u + b_i|, \; i = 1, 2, \ldots m,$$

yields the equivalent (constrained) optimization problem:

$$\begin{aligned} &\min_{u \in I\!R^n,\, v \in I\!R^m} \sum_{i=1}^{m} v_i, \\ &\text{subject to} \quad v_i = |a_i^T u + b_i|, \qquad i = 1, 2, \ldots m. \end{aligned} \tag{7}$$

If one can show that this is equivalent to

$$\begin{aligned} &\min_{u \in I\!R^n,\, v \in I\!R^m} \sum_{i=1}^{m} v_i, \\ &\text{subject to} \quad v_i \geq |a_i^T u + b_i|, \qquad i = 1, 2, \ldots m, \end{aligned} \tag{8}$$

then the proof is over, since problem (8) is simply the linear programming problem:

$$\begin{aligned} &\min_{u \in I\!R^n,\, v \in I\!R^m} \sum_{i=1}^{m} v_i, \\ &\text{subject to} \quad -v_i \leq a_i^T u + b_i \leq v_i, \qquad i = 1, 2, \ldots m. \end{aligned}$$

It remains only to show that when we increased the set of feasible solutions (when we replaced problem (7) with problem (8)), we did not introduce any new candidate (feasible for (8) and not for (7)) which yields a strictly better objective function value. In order to see this, we shall show that an optimal solution for problem (8) necessarily satisfies the constraints of problem (7). Indeed, let us assume that u^*, v^* is an optimal solution for problem (8). If u^*, v^* were not feasible for problem (7), it would mean that there is at least one index k $(1 \leq k \leq m)$ such that $v_k^* > |a_k^T u^* + b_k|$. Consider now the vector $\bar{v}$ defined by $\bar{v}_i := v_i^*$ for all $i \neq k$, and such that $\bar{v}_k := |a_k^T u^* + b_k|$. It is then straightforward to verify that $u^*, \bar{v}$ is feasible for problem (8) and has a strictly better objective-function value than u^*, v^*. This contradicts the optimality of u^*, v^*. As a consequence, we have that if u^*, v^* is optimal for problem (8), then we necessarily have

$${v^*}_i = |a_i^T u^* + b_i|, \quad i = 1, 2, \ldots m. \tag{9}$$

Hence, by introducing m new variables and adding $2m$ linear constraints, we can reduce the piecewise-linear optimization problem (6) to a simple linear programming problem.

Returning to our jacking problem, this translates in transforming the piecewise-linear formulation (4) into the linear program

$$\min_{x \in \mathbb{R}^{N_J} b,t,s \in \mathbb{R}^{N_C}} \quad \sum_{c=1}^{N_C} \left(\frac{1-\lambda}{2} b_c + \frac{1-\lambda}{2} t_c + \lambda s_c \right)$$

$$\begin{array}{lrclcll} \text{subject to} & -b_c & \leq & M_B^c(x) & \leq & b_c, & c = 1, 2, \ldots, N_C, \\ & -t_c & \leq & M_T^c(x) & \leq & t_c, & c = 1, 2, \ldots, N_C, \\ & -s_c & \leq & S^c(x) & \leq & s_c, & c = 1, 2, \ldots, N_C, \end{array}$$

where b, t, s are vectors of $\mathbb{R}^{N_C}$ whose cth components are denoted respectively b_c, t_c, and s_c. By analogy with (9), we have, for an optimal solution x^*, b^*, t^*, s^* that

$$\begin{array}{rcl} b_c^* & = & |M_B^c(x^*)|, \\ t_c^* & = & |M_T^c(x^*)|, \\ s_c^* & = & |S^c(x^*)|, \end{array}$$

for $c = 1, 2, \ldots, N_C$. Thus, physically, b_c, t_c, and s_c represent the magnitudes of the bending momentum, the torque, and the shear load respectively at the cth contact point.

To summarize, we obtained this linear programming formulation after adding the $3N_C$ optimization variables b_c, t_c, and s_c ($c = 1, 2, \ldots, N_C$), and $6N_C$ linear inequality constraints.

F. Constraints

F.1 Lower and upper bounds

A first set of constraints is specifying the maximal amount of force that can possibly be applied to each available jacking location:

$$L_j \leq x_j \leq U_j, \quad j = 1, 2, \ldots, N_J. \tag{10}$$

Typically, the lower bound, L_j is set to zero. However, one can enforce a specific jacking location j to be used by setting L_j to a positive value.

F.2 Link between discrete and continuous variables

Let us now take into account the binary variables y_j which monitor whether jacking location j is used or not. It is essential to be able to count the number of jacking locations effectively used for two reasons. First, one of our objectives is to use as few jacks as possible. Secondly, the number of jacks available for the maintenance operation is typically much smaller than the number of possible jacking locations on the aircraft. One way to do in order to enforce the amount of force x_j to vanish when jacking location j is not used is the following. One can consider replacing, in all the above expressions of the momenta and shear loads (Section II-C), every occurrence of the force variable "x_j" with "$y_j x_j$". However, this straightforward way of modeling would yield the three quantities $M_B^c(x)$, $M_T^c(x)$, and $\mathcal{S}^c(x)$ to be quadratic in the optimization variables (i.e. x and y). The disadvantage of such a model is that we would have to deal with a mixed-integer *quadratic* optimization problem objective function. Again, solving a mixed-integer *quadratic* optimization problem is computationally much more demanding than solving a mixed-integer *linear* program. Here, we can in fact again avoid the quadratic model simply by replacing the simple bound constraints (10) with the linear constraints:

$$L_j \leq x_j \leq U_j y_j, \quad j = 1, 2, \ldots, N_J. \tag{11}$$

Hence, constraints (11) enforce the logical conditions:

$$x_j > 0 \Longrightarrow y_j = 1, \quad j = 1, 2, \ldots, N_J,$$

which enable our model to count the number of jacks effectively used. We remark that constraints:

$$y_j = 1, \quad \text{for all } j \text{ such that } L_j > 0. \tag{12}$$

are simple consequences of imposing constraints (11), which means that we impose the presence of a jack at a specified (imposed) jacking location. However, adding (12) to our formulation (as redundant constraints) is useful: it reduces the number of integer variables, which has an important impact on the overall complexity.

F.3 Equilibrium equations

Among the N_J available jacking locations, there are three particular jacking locations, those near the three landing gears: that at the fore landing gear (the *front jack*), and those at the two wing landing gears (called the *L/H main jack* and the *R/H main jack*). We shall call the three corresponding jacks *main jacks*, and the jacks used for the remaining $N_J - 3$ jacking locations will be said *auxiliary jacks* (see Figure 2). The amounts of force applied with the three main jacks are not proper optimization variables. They should simply result from the amount of force (the x_j's) applied with the auxiliary jacks, and from the mass layout of the aircraft. However, the force applied with the three main jacks are subject to (upper- and lower-) bound constraints. In typical instances, the three main jacks have to be used because they guarantee the equilibrium of the aircraft (the user then simply imposes the constraint $y_j = 1$ for each of these jacking locations). The equilibrium equations, describe the relation between the amount of force at the three main jacks in terms of the amount of force at the auxiliary jacks and gravity forces:

$$\text{Momenta:} \quad \begin{cases} -\sum_{j=1}^{N_J} |Y(j)| x_j + \sum_{p=1}^{N_P} |Y(p)| W_p = 0, \\ -\sum_{j=1}^{N_J} X(j) x_j + \sum_{p=1}^{N_P} X(p) W_p = 0, \end{cases} \tag{13}$$

$$\text{Forces:} \quad -\sum_{j=1}^{N_J} x_j + \sum_{p=1}^{N_P} W_p = 0. \tag{14}$$

Each of the equilibrium equations simply expresses the fact that the bending momentum, the torque, and the shear load respectively must be null (everywhere on the aircraft and in particular at the origin of our system of axes). Since the aircraft system is isostatic (because of the presence of the three main jacks), there is necessarily a unique solution to the (3 equations–3 unknown) system of equations (13) and (14).

G. Overall formulation

In the objective function we described in Section II-E, we only took into account our first objective: minimizing residual internal stress at the control points. Remember that our secondary aim is to minimize the number of auxiliary jacks used. In order to achieve this aim, we propose proceeding iteratively. This is motivated by the fact that in practice airliners have at their disposal, at the base of maintenance, only a certain number of jacks. We first set to a certain pre-determined value, N $(3 \leq N \leq N_J)$, the initial number of jacking locations that can be used:

$$\sum_{j=1}^{N_J} y_j = N,$$

and we compute a corresponding optimal solution x^*, y^*. We then successively decrease the allowed number of jacking locations (or, equivalently, the number of jacks), N, by one and repeat the optimization process. We stop when the values of the momenta and shear load become unacceptable. In our practical tests (cf. Section III), we always observed a sudden significant increase in the optimal objective-function value when going from a certain $N = \bar{N}$ to $N = \bar{N} - 1$, which made us fixing $N = \bar{N}$. Remark that practitioners appreciate that we provide the computed values of the momenta and shear load for all the values of N such that $\bar{N} \leq N \leq N_J$, for them to make the final decision (compromise between number of jacks used and residual internal stress measured at the control points). In practice, we start with N equal to the number of available jacks (which is, typically, smaller than N_J, the number of available jacking *locations* on the aircraft.

We are thus left with the following mixed-integer linear programming problem which we solve iteratively for different values of N:

$$\min_{x\in\mathbb{R}^{N_J}\, b,t,s\in\mathbb{R}^{N_C}\, y\in\{0,1\}^{N_J}} \quad \sum_{c=1}^{N_C}\left(\frac{1-\lambda}{2}b_c + \frac{1-\lambda}{2}t_c + \lambda s_c\right)$$

subject to

$$\begin{aligned}
-b_c \leq M_B^c(x) &\leq b_c, \qquad c = 1,2,\ldots,N_C,\\
-t_c \leq M_T^c(x) &\leq t_c, \qquad c = 1,2,\ldots,N_C,\\
-s_c \leq S^c(x) &\leq s_c, \qquad c = 1,2,\ldots,N_C,\\
-\sum_{j=1}^{N_J}|Y(j)|x_j + \sum_{p=1}^{N_P}|Y(p)|W_p &= 0,\\
-\sum_{j=1}^{N_J}X(j)x_j + \sum_{p=1}^{N_P}X(p)W_p &= 0,\\
-\sum_{j=1}^{N_J}x_j + \sum_{p=1}^{N_P}W_p &= 0,\\
\sum_{j=1}^{N_J}y_j &= N,\\
L_j \leq x_j &\leq U_j y_j, \qquad j = 1,2,\ldots,N_J,\\
y_j &= 1, \qquad \text{for all } j \text{ such that } L_j > 0.
\end{aligned} \tag{15}$$

where

• the explicit expressions of $M_B^c(x)$, $M_T^c(x)$, and $\mathcal{S}^c(x)$ are given in Section II-D according to the specific location on the aircraft of the control point c, $c = 1,2,\ldots,N_C$;

• the user-defined parameter λ is a weighting parameter that allows the user to put more or less emphasis on the contribution of the shear load with respect to the momenta. For instance, when $\lambda = 0$, then only the momenta are taken into account in the objective function. If the user chooses to set $\lambda = 1$, then only the shear load will appear in the objective function. In typical instances, one will choose $0 < \lambda < 1$. As a matter of fact, we shall see in Section III that the validation of our results are relatively insensitive to variations of the value of this parameter. This tends to show that minimizing shear force is compatible with minimizing momenta.

Remember that the above objective function is split into two terms, according to whether the control point c is located on the wings (first sum) or on the fuselage (second sum), as discussed at the end of Section II-E.

III. Computational experiments

Airbus currently uses an internal software to compute the residual internal stresses corresponding to a given list of jacking locations and corresponding forces which are proposed by an experimented engineer. In order to validate our approach, we shall compare the results we obtain on a set of real problems, with results already implemented by Airbus. Table I describes the main features of these test problems. In these practical instances, the order of magnitude of N_P, the number of aircraft parts (discretized masses), is 100.

problem	date	aircraft	aircraft mass (kg)	N_C	N_J	nb of imposed jacking locations
1	03/2002	A320	34 070	4	34	3
2	03/2002	A320	37 100	10	8	3
3	05/2002	A320	31 020	4	8	3
4	03/1998	A310	50 831	3	30	4
5	11/1997	A300-B4	56 943	3	33	4
6	09/1996	A300-B4	73 598	4	28	9
7	09/1997	A300-B2	52 761	4	21	12
8	06/1997	A300-B2	61 989	2	14	6

TABLE I
Test problems

Our method was programmed in Fortran 90. Our program calls NAG's H02BBF optimization subroutine for solving our mixed-integer linear programming formulation (using branch and bound) for a decreasing sequence of integer values of N, the number of jacks used.

First, in order to appreciate the robustness of our approach with respect to specific choices of the user-defined parameter λ, we address Problems 1 to 3 with different values of the weighting parameter: $\lambda = 0.1$ and $\lambda = 0.9$. Table II summarizes our results. For comparison purposes, we report here solutions which we obtained with a number of jacks (displayed in the second column of Table II) equal to that of the actual maintenance operation performed by Airbus. The fourth columns displays the

problem	nb of jacks	λ	solution applied in practice	optimization approach	initial N
1	7*	0.1	66 916.4	2 354.6	10
		0.9	16 679.6	2 247.1	10
2	7	0.1	33 253.5	28 123.3	7
		0.9	6 697.5	4 907.1	7
3	4	0.1	8 050.0	496.4	7
		0.9	8 826.0	380.2	7

TABLE II
Weighted residual internal stresses

objective function *value*, as defined in our optimization formulation (15), of the solution that was applied in practice by Airbus. In other words, it represents the value of:

$$\sum_{c=1}^{N_C}\left(\frac{1-\lambda}{2}|M_B^c(x)| + \frac{1-\lambda}{2}|M_T^c(x)| + \lambda|S^c(x)|\right).$$

The last column of Table II displays the same quantity corresponding to the solution found by the optimization approach proposed in this paper.

The "*" on the number of jacks for Problem 1 means that the optimization approach could find a better solution than the one applied at that time for the maintenance operation, *and* with fewer jacks. Indeed, with only 4 jacks, we propose a solution with values 12 650.7 and 8 776.3, respectively for $\lambda = 0.1$ and $\lambda = 0.9$. For the illustration purposes, Figure 2 displays the positions of the $N_C = 10$ control points (given data—represented by triangles) for Problem 2. We see also the locations chosen to assign the jacks: the position of the 3 main jacks (imposed—represented by circles), and the 4 auxiliary jacks (squares). For this instance (Problem 2), the solution found by our optimization approach selected the same 7 jacking locations (among the 8 possible locations) as those chosen by Airbus for the maintenance operation. However, the *amount* of force (the x_j) applied on the jacking locations differ. As a general comment on the results of Table II, we see that strictly better values for the (weighted) residual internal stresses were obtained using the optimization approach, and this remark applies irrespective of the value of the weighting parameter.

Remark that the weighting parameter is needed because our objective function is the sum of quantities which are expressed in different physical units. Moreover, in practice, the user is even likely to desire to define different sets of weighting parameters, for the different control points c, $c = 1, 2, \ldots, N_C$. Indeed, a particular control point could naturally require less emphasis because this point is, for instance, located in an area which can support high load with weak strain. However, for the sake of better comparison in such a multi-objective context, we next present in more details numerical results on the remaining test problems, without aggregating (with particular values of weighting parameters) the values of $|M_B^c(x)|$, $|M_T^c(x)|$, and $|S^c(x)|$. Table III displays concisely the magnitudes of the bending moment, the torque and the shear load at the different control points (remember that the objective is to minimize these quantities at every control point c, $c = 1, 2, \ldots, N_C$.) The units are decaNewton (1 daN = 10 Newtons) for the $|S^c(x)|$ measures, and meters $\cdot$ decaNewton for $|M_B^c(x)|$ and $|M_T^c(x)|$. The fact that results are grouped for two different control points is simply a consequence of symmetry. In such a multi-objective context, the weighting parameter has to be tuned so as to obtain a satisfactory solution for each control point. However, our mono-objective optimization formulation still requires a value for λ to define the progress criterion (the objective function). Therefore, we chose the compromise value $\lambda = 1/3$ to guide the optimization (it roughly amounts to attempt at minimizing the sum of the absolute values of the the bending moment, the torque and the shear load).

The quality of the optimization solution for Problem 4 and Problem 6 compares favourably with that of the solution implemented at the time of the maintenance operations. The "*" on the number of jacks for Problem 5 is to pay attention here to the fact that the same quality of solution (zero value on each control point) was attained with the optimization approach, but using fewer jacks (only 5, versus 6 for the implemented solution). For Problem 7, the quality of the solutions is comparable. For Problem 8, the optimization solution is clearly uniformly (over the various control points and measures) better than the implemented solution. We do not report here the exhaustive compared solutions (the amount of force x_j assigned to each selected jacking location j). As a general comment, we notice that the solutions generally differ in some of the x_j components, and sometimes also in the y_j values as well (the selected jacking locations). This is namely the case for Problem 6 (9 out of the 10 jacking locations are common), Problem 7 (13 out of the 15 jacking locations in common), and Problem 8 (7 out of 9).

To summarize, the optimization approach always yields solutions of comparable or better quality than the solutions effectively implemented at the time of the maintenance operations. Moreover, these solutions are produced within short computation time. Precise CPU timings were not recorded in the above computational experiments, only actual ("wall-clock") durations of the computation. However, these durations never exceeded 5 seconds (less than one second was the typical case). This was considered very satisfactory for the practicians of the jacking operations at Airbus. This is not to

problem	nb of jacks	control point c	measure	solution applied in practice	optimization approach	initial N
4	8	1	$\|M_B^c(x)\|$	3 726	0	15
			$\|M_T^c(x)\|$	0	0	
			$\|S^c(x)\|$	1 043	0	
		2,3	$\|M_B^c(x)\|$	2 029	0	
			$\|M_T^c(x)\|$	8 596	0	
			$\|S^c(x)\|$	87	1 556	
5	6*	1,2,3	$\|M_B^c(x)\|$	0	0	10
			$\|M_T^c(x)\|$	0	0	
			$\|S^c(x)\|$	0	0	
6	10	1,2	$\|M_B^c(x)\|$	211	537	15
			$\|M_T^c(x)\|$	1 209	155	
			$\|S^c(x)\|$	4 034	3 630	
		3,4	$\|M_B^c(x)\|$	4 327	4 326	
			$\|M_T^c(x)\|$	2 355	2 357	
			$\|S^c(x)\|$	3 362	3 362	
7	15	1,2	$\|M_B^c(x)\|$	12 675	11 924	20
			$\|M_T^c(x)\|$	2 392	2 250	
			$\|S^c(x)\|$	5 585	5 472	
		3,4	$\|M_B^c(x)\|$	4 307	4 397	
			$\|M_T^c(x)\|$	131	116	
			$\|S^c(x)\|$	550	542	
8	9	1,2	$\|M_B^c(x)\|$	6 912	290	13
			$\|M_T^c(x)\|$	2 320	1 021	
			$\|S^c(x)\|$	2 843	1 877	
		3,4	$\|M_B^c(x)\|$	4 555	1 267	
			$\|M_T^c(x)\|$	922	402	
			$\|S^c(x)\|$	1 131	165	

TABLE III
Residual internal stresses at the control points

mention that CPU timing improvements can be achieved if one considered exploiting the fact that for two consecutive values of N (the number of allowed jacks) we are solving very similar mixed-integer linear programming problems: indeed they differ only in one component of the right-hand side vector of (15).

IV. Conclusions

We addressed in this paper the problem of assigning forces to be applied on various jacking positions in order to weaken stress at the points where a maintenance has to be performed on an aircraft. We achieved to model the aircraft maintenance jacking problem as a mixed-integer *linear* programming problem. Binary variables tell which set of jacking locations are to be used (remember that only a limited number of jacks are available for all the possible jacking locations), while continuous variables give the amount of force to be applied on each selected jacking location. The mathematical programming formulation we introduced straightforwardly enables the integration of various constraints of practical relevance, such as requiring a maximal number of jacks to be used, imposing the presence/absence of a jack at specific locations, or fixing a (range of) value(s) for the amount of force to apply at a given jacking location (for instance, in order to support an engine). We reported very encouraging computational experiments with the jacking problem for maintenance operations (historical data) on Airbus A310, A300-B4, and A300-B2 aircrafts. The optimization approach always yielded solutions of comparable or better quality than the solutions already implemented by Airbus. Above all, the acceptable computer times required to obtain *optimal* solutions on these real-life test problems showed that the approach was viable in practical situations. As a result, our methodology is currently under the process of industrial implementation at Airbus, where it can be used as a decision-analysis tool to complement and improve the current experience-based trial-and-error way of doing.

Further work could consider using a more precise model (finite elements, instead of assuming that the aircraft is made of three beams). This would be more realistic when dealing with very large aircrafts such as Airbus' A340-500, A340-600, and A380. In such cases, one deals with long and flexible aircrafts whose geometry changes as a consequence of the jacking. However, such a study would clearly involve much more computational work.

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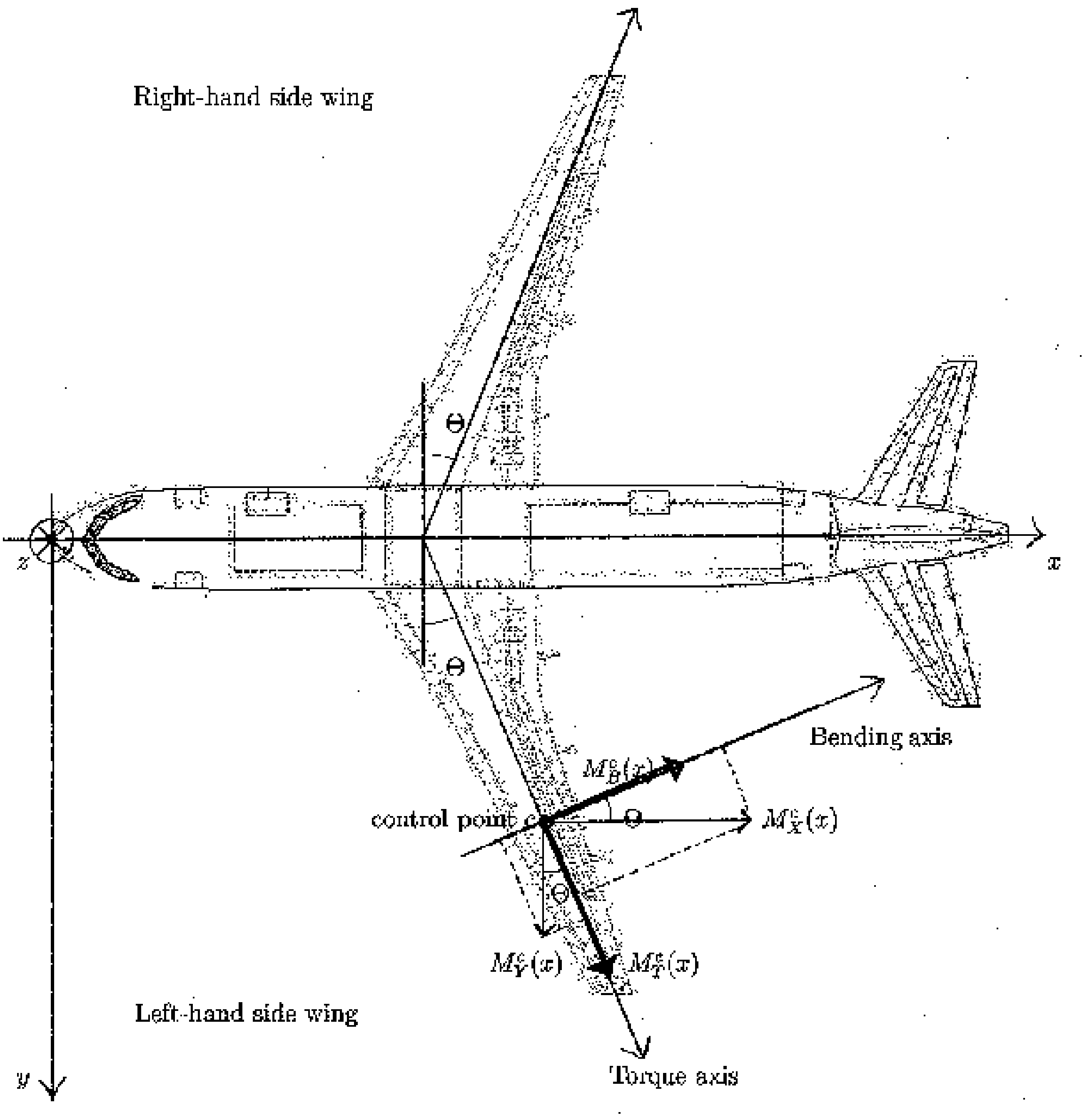

Fig. 1. Systems of axes.

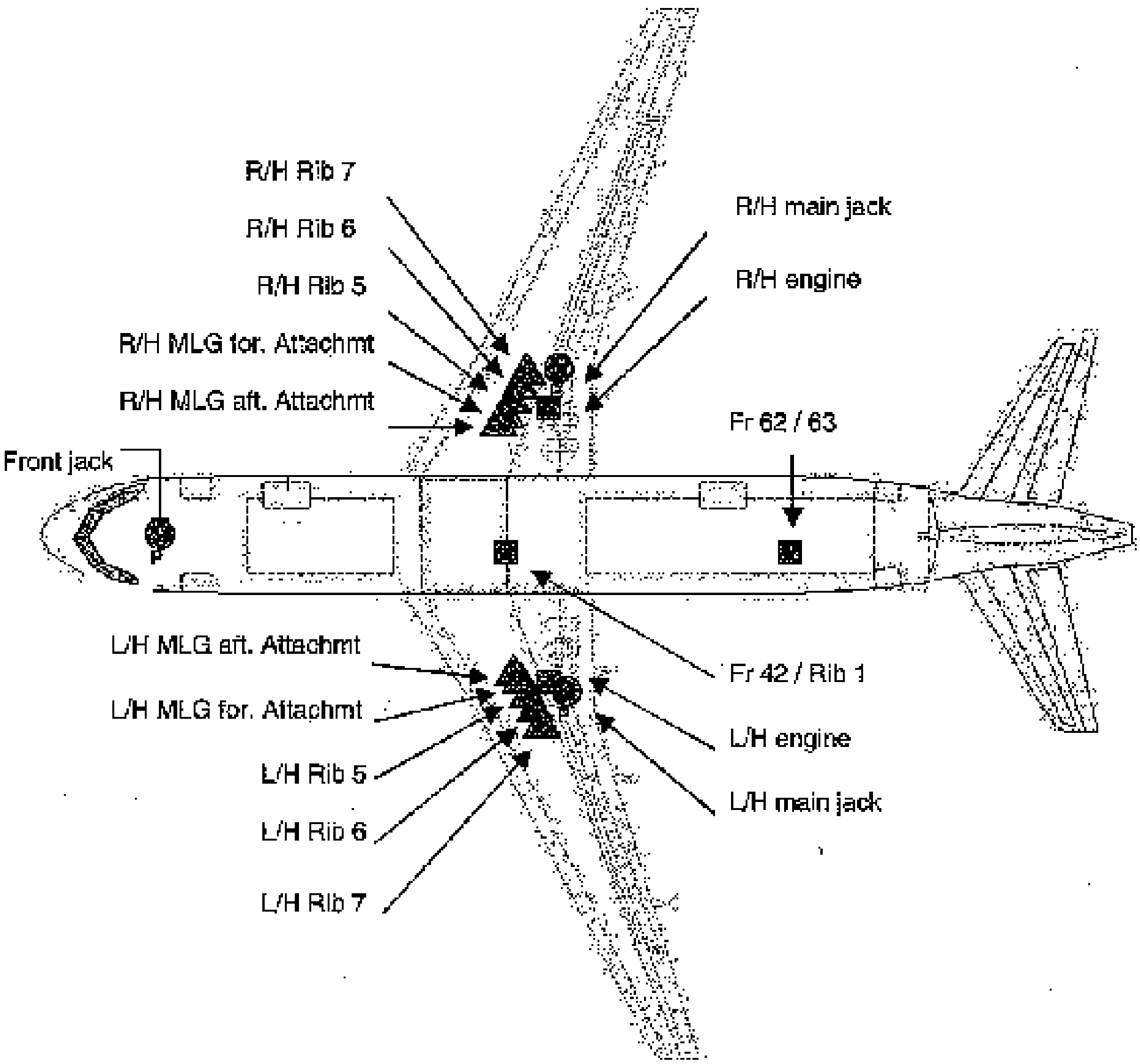

Fig. 2. Locations of the control points (△), the main jacks (◯), and the auxiliary jacks (□) for Problem 2.